

Bose Einstein Condensation as Dark Energy and Dark Matter

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We study a cosmological model in which the boson dark matter gradually condenses into dark energy. Negative pressure associated with the condensate yields the accelerated expansion of the Universe and the rapid collapse of the smallest scale fluctuations into many black holes, which become the seeds of the first galaxies. The cycle of gradual sedimentation and rapid collapse of condensate repeats many times and self-regularizes the ratio of dark energy and dark matter to be order one.

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Introduction

Recent observations including WMAP for cosmic microwave background [1] and supernovae Type Ia Hubble diagrams[2] independently indicate that some unknown dark energy (DE) component dominates the global cosmic energy density and induces the accelerated expansion of the Universe. On the other hand, previous many observations manifestly indicate that some unknown dark matter (DM) dominates the local energy density, whose total abundance turns out to be the same order as that of DE.

Amazingly without explicitly specifying dominant contents of the Universe, the standard Λ CDM model can explain many of the observational data, especially those related with the density fluctuations in the linear stage. In the non-linear stage however, the standard Λ CDM model cannot explain the very early formation of astronomical objects and re-ionization at around $z \approx 20$, which is observationally required. Some extra mechanism such as biasing is inevitable for the galaxy formation, since the gravity itself cannot sufficiently compact matter.

In this letter, we propose a unified model of DE and DM in the context of a cosmic phase transition and further the mechanism for the early formation of non-linear objects. The key feature of DE would be the volume-independent negative pressure, which guarantees the accelerated cosmic expansion through the Einstein equation. The only known matter which shows negative pressure would be some classical coherent objects such as scalar fields. As an origin of such coherent object, we specifically consider the Bose-Einstein condensation (BEC) of a boson field with attractive interaction, and we can identify the condensation as DE and the exited bose gas as DM. The pressure of BEC for ideal gas is volume independent and therefore an introduction of attractive interaction would easily yields negative pressure. If we adopt this scenario, the DE dominance means that our universe is in almost the ground state described by a macroscopic wave function. Thus the largest scale of the Universe, as well as the smallest, would turn out to be described by quantum mechanics.

Basic model we construct would have the following general features: (a)Bose gas is introduced as DM which initially dominates the energy density and the condensate of the boson is identified as DE. (b)The condensate has

negative pressure due to its potential which reflects the attractive interaction. For the spatially uniform component of BEC, this negative pressure works as a cosmological constant and guarantees the accelerated expansion. (c)The sedimentation of the condensate slowly proceeds in the cosmic evolution. (d)When the energy density of BEC exceeds some critical value, the condensates rapidly collapses into compact boson stars and black holes, which work as the standard cold dark matter and also become the seeds of galaxies. Simultaneously a new sedimentation process begins. This cycle of sedimentation-and-collapse repeats many times. These rapid collapses take place well after the photon-decoupling stage, and therefore the large scale structure predicted by the Λ CDM model and actually observed pattern in CMB fluctuations would not strongly be violated.

cosmic BEC mechanism

The BEC occurs when the thermal de Broglie length $\lambda_{dB} \equiv (2\pi\hbar^2/(mkT))^{1/2}$ exceeds the mean separation length of particles $n^{-1/3}$, where $n = N/V$ is the mean number density of the boson and m is the boson mass. The critical temperature is given by

$$T_C = \frac{2\pi\hbar^2 n^{2/3}}{m\zeta(3/2)}. \quad (1)$$

At the transition point, the chemical potential μ associated with the conservation of particle number N within the volume V shows singular behavior $\mu \rightarrow -0$. The condensation is possible only for non-relativistic stage $T < m$ and the particle number N is conserved. On this stage, the cosmic energy density behaves as

$$n = n_0 \left(\frac{m}{2\pi\hbar^2} \frac{T}{T_0} \right)^{3/2}, \quad (2)$$

where n_0, T_0 are the number density and the temperature at some moment in the non-relativistic stage. In deriving eq.(2), we have assumed the adiabatic evolution in the sense that the entropy per particle $s/n = \ln(e^{5/2}(mT)^{3/2}/(2\pi\hbar^2)^{3/2}/n)$ is conserved. The number density dependence of the temperature $T \propto n^{2/3}$ is the same as Eq.(1). Therefore the condition $T < T_C$ sets the upper limit of the boson mass; if we adopt the value $\rho_{now} = 9.44 \times 10^{-30} \text{ g/cm}^3$, then $m < 2 \text{ eV}$. Moreover,

the boson gas dominance condition in the non-relativistic stage sets the lower limit of the mass $m > 2$ eV, provided the boson field has the same temperature as that of radiation. However, if the boson field is not the thermal origin, there is no lower limit for the mass. If the above conditions hold, the BEC process starts when the boson becomes non-relativistic and it continues afterwards.

Quantum liquid model of cosmic BEC

In order to describe the dynamics of BEC, the mean-field analysis[3] using the Gross-Pitaevskii equation is usually adopted. This equation has a form of non-linear Schrödinger equation,

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \psi + V\psi + g|\psi|^2\psi, \quad (3)$$

where $\psi(\vec{x}, t)$ is the condensate wave function, $V(\vec{x})$ is the potential, $g = 4\pi\hbar^2 a/m$, and a is the s-wave scattering length. If we decompose the wave function as $\psi = \sqrt{n}e^{iS}$, and define the velocity as $\vec{v} = \hbar\vec{\nabla}S/m$, then Eq.(3) reduces to the continuity equation, and the hydrodynamic equation,

$$m \frac{\partial \vec{v}}{\partial t} + \vec{\nabla} \left(\frac{mv^2}{2} + V + gn - \frac{\hbar^2}{2m\sqrt{n}} \Delta \sqrt{n} \right) = 0, \quad (4)$$

except the last term which is quantum origin ($\propto \hbar^2$). This term can be neglected if the wave number of the mode k satisfies $k^2 < k_c^2 \equiv 2mgn/\hbar^2$; i.e. large scale mode. Further, if we choose the attractive interaction ($g < 0$), BEC can be described as fluid with negative pressure. This type of argument on BEC yields the description by general liquid with the equation of state $p = -A\rho^\alpha$.

In this letter, we introduce the following simplest model for the BEC: Boson gas is identified as cold DM with the equation of state $p = 0$, and the condensate as DE with $p = -\rho$. The sedimentation of BEC in the uniform expanding Universe slowly proceeds with the time scale Γ^{-1} . This setting is very similar to the chaplygin gas model[4] with the equation of state $p = -A/\rho$, except that in the latter, DE and DM properties are simultaneously included in this single equation of state of a single phase.

Since the Universe is initially extremely uniform, this condensation would also be uniform. The energy density of the excited bose gas is diluted by the cosmic expansion, however, that of condensate is not diluted. This is because the work supplied to expand the volume V to $V + dV$ is $-pdV = \rho dV$, which is exactly the necessary and sufficient amount of energy to produce the new condensate in the region dV with the same energy density. Therefore eventually the condensate would dominate the excited gas component and the expansion law of the Universe changes from the decelerated expansion to the accelerated expansion.

In the early stage when the bose gas density dominates that of condensate, density fluctuations are controlled by the dominant component, i.e. the bose gas, and their

evolution is described by the standard Λ CDM model. However in the later stage when the condensate density dominates the bose gas density, the situation drastically changes. The linear perturbation equation for the gauge invariant quantity, in our case of the equation of state $p = -\rho$, reduces to

$$\delta_k'' + 5\delta_k' = - \left(6 - \left(\frac{k}{aH} \right)^2 \right) \delta_k, \quad (5)$$

where $(\dots)' \equiv \frac{d(\dots)}{d \ln a}$, and $\delta_k \equiv \delta\rho(k)/\rho$. According to this, a small scale mode $\tilde{k}^2 > 6H^2$ rapidly grows, and an almost cosmic horizon scale mode $\tilde{k}^2 < 6H^2$ slowly decays, where the comoving wave number $\tilde{k} \equiv k/a$ is defined. Moreover, the smaller the fluctuation scale, the faster the growing process: $\delta_k \propto \exp(t\tilde{k}/H)$. Note that this rapid collapse is related with the fact that in the gas of negative pressure, there is no sound wave ($c_S^2 < 0$). The situation now considering is not the (never growing) density fluctuations in the de Sitter space in which the pressure is a strict constant.

Let us consider the non-linear stage of the condensate collapse. We suppose a uniform spherically distributed over-density region of BEC of the radius r and the density ρ . Since the pressure gradient only exists on the surface of the sphere, the surface is isotropically compressed to form a dense skin. In this process, the skin region with the width dr acquires energy $4\pi r^2 dr (-p) = 4\pi r^2 dr \rho$ which is exactly the mass of the skin. Therefore the skin soon acquires the light velocity. The skin itself has large negative pressure in magnitude and therefore self-focusses. Since this skin is still located at the same pressure gradient, it is further compressed and eventually wipe up the whole condensate toward the center. This collapsing process can be expressed in the evolution equation of the skin radius,

$$\frac{d(m_t \gamma \dot{r})}{dt} = -4\pi r_t^2 \rho, \quad (6)$$

where $m_t = (4\pi/3)(r_0^3 - r_t^3)\rho$ is the time dependent mass of the skin, and the right hand side of Eq.(6) is the total force acting on the skin $4\pi r_t^2 p$. All solutions of Eq.(6) approach to the constant velocity solution

$$\gamma = \sqrt{2}, \dot{r} = 1/\sqrt{2}. \quad (7)$$

Thus the collapse of the condensate is almost the light velocity.

Self gravity of the sphere would further accelerate the skin collapse especially in the later stage. The collapse would continue until the Heisenberg uncertainty principle begins to support the structure, or a black hole is formed, or it bounces back outward if the condensate melts at the final stage of the collapse. Anyway the collapsed condensate forms localized compact objects classified as cold dark matter.

Gradual sedimentation of the condensation— Self Organized Criticality

We now turn our attention to the global evolution of DE/DM in the expanding Universe. The evolution of the various energy densities are governed by the set of equations,

$$\begin{aligned} \rho &= \rho_c + \rho_g + \rho_l, & H \equiv \frac{\dot{a}}{a} &= \sqrt{\frac{8\pi G\rho}{3}}, \\ \dot{\rho}_c &= \Gamma \rho_g, & \dot{\rho}_g &= -3H\rho_g - \Gamma \rho_g, & \dot{\rho}_l &= -3H\rho_l, \end{aligned} \quad (8)$$

where ρ_c , ρ_g , and ρ_l are the energy densities of condensate, excited boson gas, and the localized energy density after the rapid collapse, respectively.

This set of evolution equations are valid when the condensate does not dominate the energy density: $\rho_c < \rho_g + \rho_l$. Once it dominates $\rho_c > \rho_g + \rho_l$ after the time scale Γ^{-1} at around $z = z_c$, inhomogeneous components of the condensate would rapidly collapse, and some fraction of ρ_c is transformed into ρ_l . Then the condition $\rho_c < \rho_g + \rho_l$ is recovered and the gradual sedimentation of the condensate proceeds again following Eq.(8) during the time scale Γ^{-1} . This repeated “chase and collapse” process by DE(ρ_c) and DM($\rho_g + \rho_l$) self-regularizes the ratio of them to fix order unity: $\rho_c \approx \rho_g + \rho_l$. This kind of autonomous dynamics designated as Self Organized Criticality (SOC) is widely known and observed in nature in various places[5].

In the late stage when the Universe is locked in this SOC phase, let us approximate $\rho_c = \rho_g + \rho_l$. Then the Einstein equation $\ddot{a}(t) = -(4\pi G/3)(\rho + 3p)a(t)$, $(\dot{a}/a)^2 = (8\pi G/3)\rho$ has the solution

$$a(t) \propto t^{4/3}, \quad \rho(t) \propto a(t)^{-3/2} \quad (9)$$

which corresponds to the deceleration parameter $q = -\ddot{a}a\dot{a}^{-2} = -1/4$.

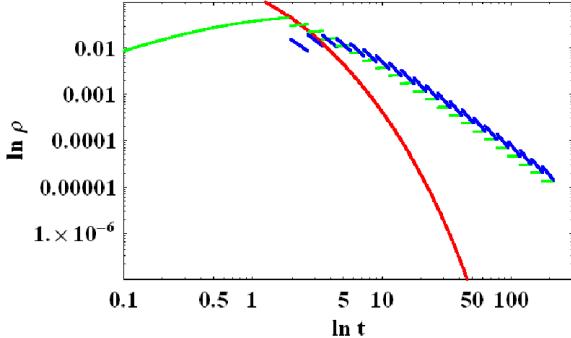


FIG. 1: Time evolution of various densities. Green= ρ_c , Red= ρ_g , and Blue= ρ_l . We set the parameter $\Gamma = 0.1$ in the unit of $8\pi G/3 = 1$, and the transition rate from ρ_c to ρ_l is set to be $1/3$. After 22 cycles of ρ_c dominance, the scale factor has evolved about 200 times. DE(ρ_c) and DM($\rho_g + \rho_l$) self-regularize their ratio to be of order unity.

Predictions and Observational tests of the model

We now examine how our model is checked by observations.

(a) Power spectrum of the density fluctuations in the linear regime The collapse of the condensate proceeds in the smallest scale. This is because the density fluctuation is stronger in the smaller scale, and the collapse proceeds with almost the light speed. Therefore the BEC model does not affect the power spectrum in the linear (large scale) regime however it extremely enhances the smaller non-linear regime. Especially the many black hole formation from DE in the very early stage is the most prominent characteristic of our model. Such black holes gradually cluster in much larger scale along the standard scenario of Λ CDM model.

(b) Power spectrum of CMB The condensation is supposed to dominate well after the decoupling time ($z_c < 1000$ or $\Gamma^{-1} > 4 \times 10^5$ year). Therefore the temperature inhomogeneity generated within the decoupling era would not be changed from the standard Λ CDM model. However the integrated Sacks-Wolfe effect, which originates from the non-uniform gravitational potential after the decoupling time, has a chance to modify the spectrum. In our BEC model, strong non-linear structure forms very rapidly in the smallest fluctuation size, which is supposed to be well outside of the linear regime.

(c) Void-wall structure The collapse of the condensate repeats many times and the smallest scale fluctuations dominantly collapse in each process. Although in the earlier collapsing process the smallest scale fluctuations (ex. galaxy scale) dominantly forms non-linearity, in the later collapsing process fairly larger scale fluctuations, much larger than the super clusters, have a chance to dominantly form non-linearity. Moreover in the earlier collapsing process almost spherically symmetric objects would be preferably formed and therefore the non-linearity is almost point-like. On the other hand in the later collapsing process, since the scale is large, much irregular fluctuations would dominantly collapse and therefore the non-linearity can be plain-like. This difference comes from the fact that the collapse is triggered by the pressure which works on surface and not by gravity which works on volume. This plain-like non-linearity may characterize the largest scale structure of the Universe.

(d) A first star harbors a boson star. In our model, once the condensate dominates the comic density, vary rapid collapses occur especially in the smallest scale of density fluctuations. Associated with this process, the collapsing object can easily fragment into many pieces because the pressure is always negative. Therefore we can expect the large amount of collapsing objects. For bosons, only the Heisenberg uncertainty principle can support the structure against the collapse. This is the quantum pressure expressed in the last term of Eq.(4). This structure is known as the boson star[6]. The size R of the object is order of the compton wave length: $\lambda_{compton} = 2\pi\hbar/(mc) \approx 2R$. This size must be larger than the Schwarzschild radius $R > 2GM/c^2$ for this object not to collapse into a black hole. These equations

yield the critical mass for the boson star.

$$M_{critical} \approx m_{pl}^2/m \equiv M_{KAUP}, \quad (10)$$

only below which a structure can exist. For example, $m = 10^{-5}$ eV yields the critical mass about the Earth mass: $M_{critical} = 10^{-5} M_\odot = M_\oplus$. These compact structures may have captured in the process of the first star formation. However even this is the case, such seed boson star and the condensate should be melted into boson gas in the high temperature stellar center, leaving no detectable relic in principle.

(e) A first galaxy harbors a giant black hole. If the fluctuation mass is larger than the critical mass $M > M_{critical}$, then the collapse inevitably continues until a black hole is formed. In this process, since $p < 0$, $dV < 0$, no heating is expected thermodynamically. However the gravitational energy released in this collapse would be GM^2/R , and if this amount of energy is used to heat up the condensate, then the temperature would be, from $NT \approx GM^2/R$ and $R \approx GM$, $T \approx GMm/R \approx m$. Precisely at this point the boson becomes relativistic $T \approx m$ and the particle number no longer conserves. Then the chemical potential is not well defined and the condensation melts into the ordinary gas with positive pressure. Therefore the boson gas stops collapse and violently expands outward. In this process, some fraction of the condensate would form a central black hole and the rest of the condensate would melt into the ordinary gas and expand outward. The gravitational potential of this structure attracts baryon to form a cluster around the central black hole. If the size is appropriate there forms a galaxy, which harbors a black hole in the center and the boson gas and baryon in the outskirts.

What would happen for the expanding boson gas? The expansion of once melted boson gas keeps the relation $T \approx GMm/R$, $\rho \approx M/R^3$, and therefore $T \propto \rho^{1/3}$. Since this temperature is still below the critical temperature, the boson gas would eventually condense and re-collapse. Then the condensation melts and the boson expands again. Apparently this bounce repeats multiple times with dissipation until the gas thermalizes completely.

Since the strong non-linearity is formed in the very early stage at around $z = z_c$, no extra biasing process is necessary in our model. In this sense our model is strongly bottom-up type. Early formed stars and galax-

ies should be the origin of the re-ionization of the Universe around $z = z_c$.

(f) Ejection of matter The above bounces of the condensate need not to be isotropic. Especially when the total angular momentum is not vanishing, the bounce would preferably strong in the direction of the angular momentum. This is because the centrifugal force naturally forms dilute cone region in the direction of angular momentum. Therefore we naturally expect, for each bounce, that a pair of blobs of the condensate is ejected with relativistic velocity parallel to the angular momentum. The blobs drag baryon which generally emits radiation. The ejection of blobs is expected to takes place repeatedly many times.

If the ejection period is sufficiently small and the aligned blobs are simultaneously visible, then this may be observed as two jets emanating from the center of the galaxy toward the direction parallel to the angular momentum. The essential characteristic of our model is that the created jets are all discrete, i.e. sequence of high-speed blobs, and not simply a continuous flow of plasma gas. Although at present we cannot quantitatively describe such highly non-linear processes, we can naturally expect the firm positive correlation among the mass of the central black hole, the size of the galaxy, and the mean separation of the ejected blobs.

Summary and Further Developments

We have developed a unified cosmological model of DE and DM using the Bose-Einstein condensation of a light boson. Fast formation of compact objects and the self organized critical Universe are realised.

In the context of BEC, there are many laboratory experiments using alkaline atom gas, in which almost ideal BEC is realized[7]. Especially the collapse dynamics of BEC with negative pressure [8] [9] may be useful to develop our cosmological BEC model. It would be interesting if the Gross-Pitaevskii equation supplemented by the dissipative term, which represents the transition between the condensate and excited gas, would lead to the quasi-periodic oscillation of BEC collapse. We also have to consider the coherence of BEC from various aspects[3]. These issues are now under investigation.

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